

Coherent backscattering of light by cold atoms: Theory meets experiment

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Abstract. – Coherent backscattering (CBS) of quasi-resonant light by cold atoms presents some specific features due to the internal structure of the atomic scatterers. We present the first quantitative comparison between the experimentally observed CBS cones and Monte Carlo calculations which take into account the shape of the atomic cloud as well as the internal atomic structure.

When light is elastically scattered off an optically thick disordered medium, the interference between all partial waves produces strong angular fluctuations of the intensity distribution (speckle pattern). Averaging over the positions of the scatterers washes out interferences and produces a smooth reflected diffuse intensity, *except* around backscattering where it is enhanced. This coherent backscattering effect (CBS) originates from a two-wave interference, namely the interference between waves traveling along the same scattering paths but in reverse order [1, 2]. This interference is constructive at exact backscattering and only survives, after averaging, in a narrow angular range $\Delta\theta \simeq 1/k\ell$ around it (k is the light wave number and ℓ the scattering mean free path). The CBS enhancement factor measures the ratio of the maximum intensity (measured at backscattering) to the background intensity measured at angles $\theta \gg 1/k\ell$. Because it originates from a two-wave interference, the enhancement factor is bounded by 2.

For vector waves like light, either linear or circular polarization can be considered. This leads to four different polarization channels in scattering experiments: $lin \parallel lin$ where both the incoming and outgoing photons are linearly polarized along the same axes, $lin \perp lin$ where they are linearly polarized along orthogonal axes, $h \parallel h$ where they are both circularly polarized with the same helicity (because they propagate in opposite directions, they have opposite polarizations) and $h \perp h$ where they are circularly polarized with opposite helicities, *i.e.* same polarization.

For spherically symmetric scatterers (and hence for point-dipole scatterers) and in the $h \parallel h$ channel the CBS enhancement factor is equal to 2 [3] because the following conditions are met: i) All scattering paths have a distinct reverse counterpart; ii) The two paths in each pair contribute with the same amplitude and phase, so that a maximum interference contrast is guaranteed. Condition i) is not true for single scattering paths which thus do not contribute to CBS. In general, this implies an enhancement factor slightly smaller than 2. However, for spherically symmetric scatterers, there is no single-scattering signal in the backward direction in the $lin \perp lin$ and $h \parallel h$ channels. Condition ii) is met in the parallel channels $h \parallel h$ and $lin \parallel lin$ as a general consequence of reciprocity [4] (which is equivalent to time-reversal symmetry in the absence of absorption). Therefore, an enhancement factor much smaller than 2 has only been observed in the perpendicular channels or by breaking the reciprocity with the help of an external magnetic field [5].

The recent experimental observation of CBS of light by an optically thick sample of laser-cooled rubidium atoms [6, 7] reported—in all channels—surprisingly small enhancement factors, typically between 1.05 and 1.2. A qualitative explanation of the experimental results in the double-scattering picture has been given in [7–9]. The rubidium atoms, being much smaller than the optical wavelength, are point-scatterers, but do not behave like classical dipoles: the atomic Zeeman internal structure leads to a violation of the previous conditions for a perfect CBS enhancement. Condition i) is not met because single scattering is present in all polarization channels, due to transitions between different atomic sub-states. Condition ii) is not met because the amplitudes associated with the direct and reverse scattering path are in general *not equal*. This imbalance of the amplitudes reduces the interference contrast, and leads to a CBS cone much smaller than for point-dipole scatterers.

In this paper, we present the first extensive comparison between the experimentally measured enhancement factors and CBS cone shapes (in all four polarization channels) and a theoretical calculation which takes into account the two most important ingredients of the experiment: the atomic internal structure and the peculiar shape of the scattering medium, an approximately spherical cloud of cold atoms produced in a magneto-optical trap (MOT).

The experimental setup is described in detail in [7]. The probe laser beam is tuned on the $D2$ line $5S_{1/2} \rightarrow 5P_{3/2}$ of Rb^{85} at $\lambda = 780$ nm and is resonant with the corresponding hyperfine ($F = 3 \rightarrow F' = 4$) transition (natural linewidth $\Gamma/2\pi = 5.9$ MHz, on-resonant light scattering cross-section $\sigma = \frac{(2F'+1)}{3(2F+1)} \frac{3\lambda^2}{2\pi}$). An atomic cloud, containing about $\mathcal{N} = 7 \times 10^9$ atoms, is produced in a MOT at a temperature in the $100 \mu\text{K}$ range (residual rms velocity spread about 10 cm s^{-1}). Imaging techniques show that atoms in the cloud are distributed with a quasi-Gaussian density of FWHM (x,y,z) -dimensions $5.88 \text{ mm} \times 4.87 \text{ mm} \times 4.63 \text{ mm}$ (axis y is the polarization axis of the CBS probe light in the linear channels and axis z its propagation axis). The on-resonance mean free path at the center of the sample $\ell = 1/(n_0\sigma)$ is of the order of $200 \mu\text{m}$, much larger than the wavelength (n_0 is the atomic density at the trap center). We are thus in the dilute regime where $k\ell \gg 1$. The typical width of the CBS cone is of the order of 0.3 to 0.9 mrad, sufficiently above the resolution limit of the apparatus (0.1 mrad).

The numerical calculation of the CBS cone takes into account the internal structure of the atom. Since all possible hyperfine dipole transitions for the $D2$ line are well separated on the scale of the linewidth Γ , we only consider quasi-resonant light scattering induced by the above-mentioned *closed* hyperfine transition. We indeed expect it to give the dominant contribution to scattering (the nearest transition $F = 3 \rightarrow F' = 3$ lies 20Γ away). Note, however, that the contribution of other transitions is currently under investigation [10]. When an atom scatters the incoming light, it may stay in the same Zeeman sub-level—this is a Rayleigh transition—or change its magnetic quantum number—this is a degenerate Raman transition. In both

cases, at weak laser intensities and if recoil and Doppler effects are negligible (which one expects to be the case for our cold atomic cloud), the scattered photon has the same frequency as the incoming photon: the scattering is *elastic*. The scattering amplitude by a single atom depends on the initial and final Zeeman sub-levels, on the scattering direction and on the incident and scattered polarizations [9]. As the atoms produced in a MOT are not in well-defined internal states, but rather in a statistical mixture of Zeeman states, the calculation of the CBS cone requires, in addition to the usual position averaging, an averaging over the possible internal ground states of the atom. We choose to perform the average over the positions of the scatterers with a Monte Carlo method and to use an internal analytical average by employing the average atomic scattering vertex [9, 11]. The details of the method are given in [12]. The essential assumption is that all Zeeman sub-levels in the ground state are equally populated, without any coherence between sub-levels. This is a reasonable assumption provided no optical pumping takes place in the medium (see the discussion below). Our Monte Carlo method is flexible, as it makes it possible to compute the CBS cone for an arbitrary spatial repartition of the scatterers. Furthermore, it allows us to take into account some rather small, yet not negligible effects, such as the non-uniform incoming intensity sent on the sample (because a Gaussian laser beam and diaphragms are used). It is important to note that all the parameters entering the numerical calculations (optical thickness of the medium, shape and dimensions of the atomic cloud, geometrical properties of the incoming laser beam) have been experimentally *measured* which means that the CBS cones that we calculate have *no* adjustable parameter.

Figure 1 shows the CBS cones recorded for an atomic cloud of optical thickness $b = \sqrt{2\pi}r_0/\ell = 26$ in the four polarization channels, compared with the cones computed using our Monte Carlo method (r_0 is the rms radius of the atomic cloud along the z -axis). The plotted cones are an angular average of the 2D images recorded on the CCD. This data processing improves the signal-to-noise ratio at the expense of smearing out any angular anisotropies, either due to the scattering itself or to the shape of the scattering medium. In the circular polarization channels, this proves to be safe since the recorded cones are isotropic (although the scattering medium itself has not a perfect cylindrical symmetry around the probe propagation axis). In the $lin \parallel lin$ channel, the cone shape is anisotropic (roughly an elliptical shape, see table I) and the widths are different along the direction of the incoming polarization and perpendicular to it [13]. In the $lin \perp lin$ channel, the CBS cone has a fourfold symmetry [13] and two different widths can be measured, either along one of the polarization axes or at 45° from it. The FWHM angular widths have been measured in the various configurations and are displayed in table I, together with the result of the Monte Carlo calculation.

For the enhancement factor, the most dramatic effect of the atomic internal structure, already noticed in the first experimental results [6, 7], is that the “best” channel for point-dipole scatterers ($h \parallel h$) is the worst one for atomic scatterers. Conversely, the worst channel for point-dipole scatterers ($h \perp h$) is the best one for atomic scatterers. This has already been established in [8] and further justified in [9] by carefully analyzing the atomic scattering vertex. But, as a consequence of the approximations done (analysis limited to single and double scattering in a semi-infinite medium with constant atomic density), the CBS cone computed in [8] had only roughly the correct angular width (and the mean free path had to be adjusted) while its shape was not in excellent agreement with the experimental result, especially in the wings. Moreover, the enhancement factors were only fairly well reproduced by the calculation.

The present agreement is much more satisfactory because it takes into account both higher-order scattering and the geometrical effects induced by the peculiar shape of the medium. The cone shapes are very well reproduced as well as the angular widths. Note that the wings of the cones are well reproduced and that non-trivial details such as the fact that the CBS cone

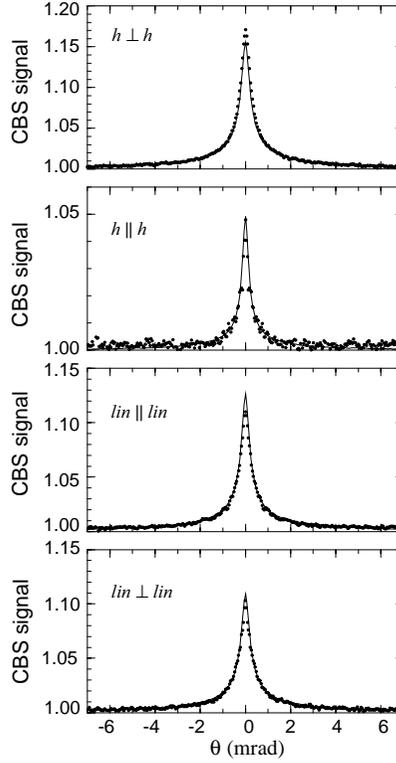


Fig. 1 – The CBS cones experimentally observed on a quasi-spherical cloud of cold rubidium atoms (optical thickness 26) illuminated by a weak resonant laser beam, in the four polarization channels. The CBS cones have been angularly averaged in order to improve the signal/noise ratio. The solid lines are the theoretical predictions of a Monte Carlo calculation taking into account the internal atomic structure, the shape of the scattering medium, and the geometry of the incoming laser beam. All the parameters are experimentally *measured*, so that there is *no* adjustable parameter. The shapes and angular widths of the various cones are very well reproduced, including the wings of the cones and details like the small angular width in the $h \parallel h$ channel.

TABLE I – Enhancement factor α and angular width $\Delta\theta$ (FWHM in mrad) experimentally observed in the various polarization channels (exp), compared to the results of our Monte Carlo calculation (th) without any adjustable parameter. “scanpar” and “scanperp” refer to the widths measured in the direction parallel to the incoming polarization, respectively perpendicular to it. The experimental uncertainty on the widths is ± 0.03 mrad, while the enhancement factors are measured ± 0.006 (± 2 standard deviations).

| Channel | α (exp) | α (th) | $\Delta\theta$ (exp) | $\Delta\theta$ (th) |
|------------------------------|----------------|---------------|----------------------|---------------------|
| $h \perp h$ | 1.171 | 1.156 | 0.61 | 0.58 |
| $h \parallel h$ | 1.048 | 1.049 | 0.30 | 0.40 |
| $lin \perp lin$ averaged | 1.103 | 1.109 | 0.62 | 0.58 |
| $lin \parallel lin$ averaged | 1.110 | 1.126 | 0.62 | 0.54 |
| $lin \parallel lin$ scanpar | ” | ” | 0.91 | 0.76 |
| $lin \parallel lin$ scanperp | ” | ” | 0.51 | 0.44 |

is significantly narrower in the $h \parallel h$ channel than in the $h \perp h$ one are also well predicted by our calculation. This is a strong evidence that our theoretical approach catches the most important aspects of multiple scattering of light by cold atoms. It also indicates that most of the experimental parameters are under control.

However, there are small differences for the angular widths, at most 0.15 mrad, which is a statistically significant deviation. Complementary Monte Carlo calculations, discussed in [12], show that the angular width is rather sensitive to the detailed shape of the medium, especially in the external layers of the atomic cloud. Keeping the same optical thickness, but changing the density of the medium from a Gaussian density $\exp[-r^2/2r_0^2]$ to an $\exp[-r^4/4r_0^4]$ density (*i.e.* sharper edges) increases the angular width by more than 50%, the precise value depending on the polarization channel. As the atomic density in a MOT is not precisely a Gaussian function, we attribute the small differences in the angular widths to an imperfect control of the shape of the medium.

Another noticeable difference is that the enhancement factor is slightly underestimated in the $h \perp h$ channel and slightly overestimated in the linear channels.

In fig. 2, we show, for the parameters of fig. 1, the contributions of the various orders of scattering to the background and the CBS cone peak value. The quantities are easily extracted from the Monte Carlo calculation. One can make several observations. Firstly, the background contributions decrease rather slowly with the scattering order N . For a semi-infinite medium, the standard diffusion approximation predicts that it decreases like $N^{-3/2}$ in excellent agreement with our numerical observation [14]. At very large order, the finite size of the medium implies a faster decrease (very long scattering paths unavoidably escape the medium), but this effect is negligible for orders lower than few tens. On the contrary, the contributions to the CBS cone decrease *exponentially* with N . In the case of the $F = 3 \rightarrow F' = 4$ transition, theory [15, 16] predicts a decay like $N^{-3/2} \times (19/40)^N$ in excellent agreement with our numerical results. This unambiguously proves that, for atoms with a degenerate ground state, the CBS effect is mainly dominated by low-order scattering and the properties of the atomic scattering vertex. This also explains why the CBS cone computed in [8] gives a reasonable estimate of the enhancement factor. Indeed, single and double scatterings contribute to roughly 57% of the background and 68% of the CBS cone. Forgetting higher orders is thus a rather good approximation, even if the medium is optically thick.

In order to understand the small discrepancy in the enhancement factors, we calculate the enhancement factor as a function of the optical thickness, in the four polarization channels, using the experimentally measured parameters. The result is shown in fig. 3, together with the four experimental points measured at optical thickness $b = 26$. It is clear that in this regime the computed enhancement factor depends only weakly on the optical thickness. This is because the CBS cone is mainly due to low-order multiple scattering, which takes place in the external layers of the atomic cloud (on the side of the incoming laser beam), and is thus only weakly sensitive to the deep layers and the optical thickness. Complementary calculations [12] also prove that the enhancement factor—contrary to the angular width—is not very sensitive to the detailed shape of the medium.

The relative strength of the four CBS cones is well reproduced by the calculation: the $h \perp h$ cone is the most important one, followed by the $lin \parallel lin$ then $lin \perp lin$ cones, the $h \parallel h$ one being significantly smaller. The deviation in the enhancement factor between the calculation and the observation is less than 0.02, *i.e.* the height of the cone is reproduced with a relative error smaller than 14%.

We are not completely sure of the reason of such a small discrepancy. We are inclined to attribute it to a non-uniform distribution of the atomic state over the various Zeeman sub-levels. Even after the trapping beams and the magnetic field of the MOT are switched off,

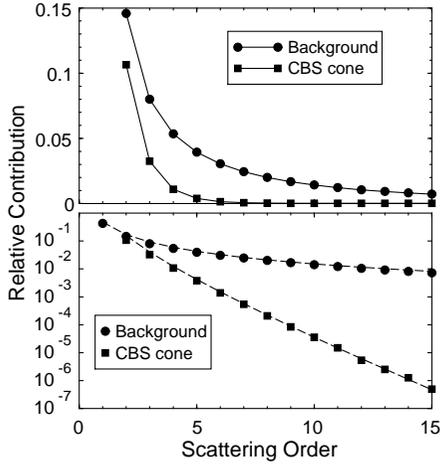


Fig. 2

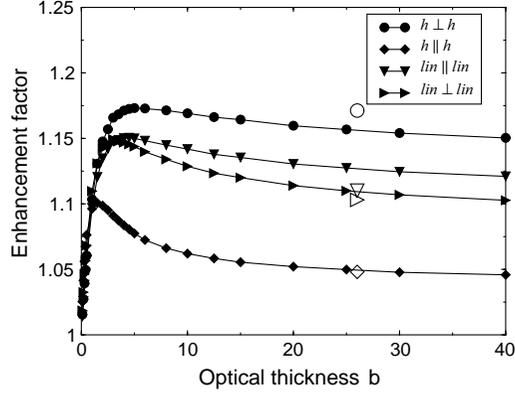


Fig. 3

Fig. 2 – Contributions of the successive multiple-scattering orders to the smooth background and to the CBS cone peak value, in the experimental conditions of fig. 1 in the $h \perp h$ channel. All contributions are normalized such that the total background is unity. The single-scattering contribution (background only) is the strongest one, 42.6% of the total. In the upper plot (linear scale), one can see the slow decrease of the contributions to the background, which shows that we are indeed in the multiple-scattering regime. The CBS contributions decrease very rapidly, which indicates that low orders of scattering are the essential contributions. On a logarithmic scale (lower plot), one clearly sees that the contributions to the background decrease algebraically with the scattering order N , roughly following the prediction of the diffusion approximation $\propto N^{-3/2}$ (dashed line). This proves that the finite geometry, which cuts the very long scattering paths, is not crucial here. The CBS contributions decrease exponentially with N , in excellent agreement with the analytic prediction using the properties of the atomic scattering vertex [16] $\propto (19/40)^N \times N^{-3/2}$, shown as a dashed line.

Fig. 3 – Enhancement factor of the CBS cone as a function of the optical thickness of the atomic cloud, computed for the experimental parameters, in the various polarization channels (the points at optical thickness 26, corresponding to data in fig. 1 and table I are not shown for clarity). The experimental points (open symbols) are in good agreement (relative error in the heights smaller than 14%), with the $h \parallel h$ channel giving the largest cone, followed by the $lin \parallel lin$ and $lin \perp lin$ channels, the $h \perp h$ cone being significantly smaller.

some polarization of the atomic ground state may remain. Another possibility —more likely in our opinion— is that the CBS probe beam, although weak, induces some optical pumping. This effect is difficult to estimate in optically thick media, because the atoms are exposed not only to the incoming beam but also to the light scattered by other atoms. The importance of optical pumping seems to be supported by preliminary experimental results which suggest that the discrepancy between the calculation and the experiment is reduced when the number of exchanged photons is decreased.

To summarize, we have presented in this paper the first comparison between experimentally observed CBS cones on a cloud of cold atoms and Monte Carlo simulations including the atomic internal structure and the peculiar sample geometry, without any adjustable parameter. The agreement is very good for the cone shapes and angular widths, as well as for the enhancement factor.

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