Thermal breakdown of coherent backscattering: A case study of quantum duality

C. Wickles and C. Müller

Physikalisches Institut, Universität Bayreuth - 95440 Bayreuth, Germany

Abstract. – We investigate coherent backscattering of light by two harmonically trapped atoms in the light of quantitative quantum duality. Including recoil and Doppler shift close to an optical resonance, we calculate the interference visibility as well as the amount of which-path information, both for zero and finite temperature.

Introduction. – A wealth of information about the motion of microscopic particles can be gathered by scattering a well-controlled probe, typically light in the case of atoms with optical resonances. Young's historic double-slit experiment demonstrated that the probe light wave shows interference if it can propagate along more than a single path. Interference fringes therefore can give sensitive information about the scatterers, as was shown by Eichmann and coworkers [1] by realizing Young's double slits with two trapped atoms. On the other hand, the motional state of the probed particle gives information about the probe field, as recently discussed by Eschner [2]. As a commonly accepted rule, one can have either full interference contrast or full which-way information at the same time. Between these two extremes, there are interesting intermediate situations that can be quantified using general quantum duality relations derived by Englert [3] that have been used, for instance, in atom interferometers [4]. In mesoscopic samples of weakly disordered clouds of cold atoms, coherent multiple scattering of light leads to enhanced backscattering [5,6].

In this letter, we investigate coherent backscattering (CBS) of light by two atoms that are trapped in harmonic oscillators. This model system can describe how atomic motion destroys multiple-scattering interference via recoil and Doppler effects. We calculate the CBS interference visibility both at zero and finite temperature in shallow traps that allow to treat the limiting case of freely moving atoms. The CBS double scattering geometry realizes a two-way interferometer where the two mutually exclusive alternatives are the order in which the photon visits both atoms. Which-way information is then present if one knows which of both atoms has been visited first. We calculate the which-way distinguishability proposed by Englert and show how it is physically encoded.
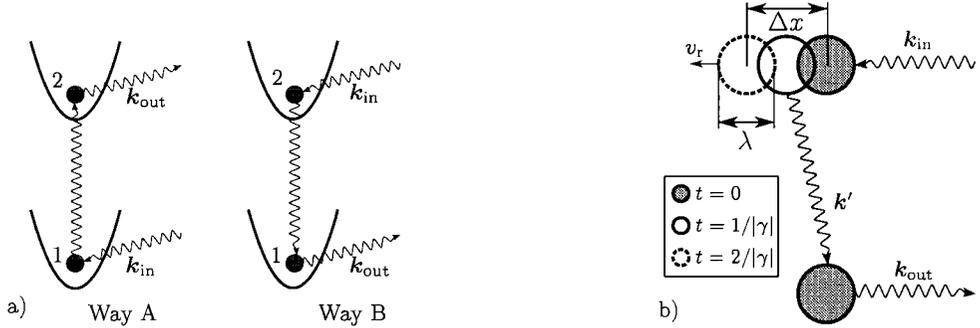
Fig. 1 – (a) Two harmonically trapped atoms backscatter a probe photon along way A or B. (b) Recoil mechanism: which-way information can be extracted if the wave-packet displacement $\Delta x = v_{\rm rec}\Delta t$ of the first scatterer during the total scattering time Δt is larger than the initial position uncertainty λ.

The setting. – Consider two atoms trapped in identical harmonic oscillators at fixed positions well separated from each other by many wavelengths of the probe light. A single incident photon with wave vector $\boldsymbol{k}_{\rm in}$ and polarization $\hat{\boldsymbol{\epsilon}}_{\rm in}$ is then scattered by the atoms and detected in the backscattering direction $\boldsymbol{k}_{\rm out} = -\boldsymbol{k}_{\rm in}$ with polarization analysis in the helicity-preserving channel $\hat{\boldsymbol{\epsilon}}_{\rm out} = \hat{\boldsymbol{\epsilon}}_{\rm in}^*$. A photon scattered in the backscattering direction by a single atom with a non-degenerate ground state has the same polarization, but opposite helicity, and does not contribute to the detected intensity. Since the atoms are far from each other, the probability for repeated scattering is very small. The detector then receives a photon that has been scattered exactly once by each atom, either along way A or along way B, see fig. 1. The total amplitude is the coherent superposition of both amplitudes. Such an interference of counter-propagating multiple scattering amplitudes in the backscattering direction is known as coherent backscattering (CBS) [5].

Without interaction, the free electromagnetic field and the internal and external (motional) atomic degrees of freedom are described by the Hamiltonian $H_0 = H_{\rm ph} + H_{\rm int} + H_{\rm ext}$ or

$$H_0 = \sum_{\mu} \hbar\omega_{\mu} a_{\mu}^{\dagger} a_{\mu} + \hbar\omega_0 \left({\rm P}_{\rm e1} + {\rm P}_{\rm e2}\right) + \hbar\omega_{\rm ho}(N_1 + N_2). \tag{1}$$

The first term constitutes the standard Hamiltonian for free photons [7]. ${\rm P}_{{\rm e}i} = \sum_{m_i} |1m_i\rangle\langle 1m_i|$ is the projector onto the degenerate multiplet $J_{\rm e} = 1$ of excited states with transition frequency ω_0 from the non-degenerate ground state $J_{\rm g} = 0$ of atoms $i = 1, 2$. The atomic motion is described by the number operators $N_i = N_{ix} + N_{iy} + N_{iz} = \boldsymbol{a}_i^{\dagger} \cdot \boldsymbol{a}_i$ of excitations in the three-dimensional isotropic harmonic oscillators with frequency $\omega_{\rm ho}$.

The dipole interaction $V = -\boldsymbol{D}_1 \cdot \boldsymbol{E}(\boldsymbol{R}_1) - \boldsymbol{D}_2 \cdot \boldsymbol{E}(\boldsymbol{R}_2)$ couples the photon field both to the electronic states and the motional degrees of freedom. $\boldsymbol{D}_i$ is the electronic dipole transition operator for atom i. The electric field operator $\boldsymbol{E}(\boldsymbol{R})$ is evaluated at the atomic center-of-mass position $\boldsymbol{R}_i = \boldsymbol{R}_i^{(0)} + \boldsymbol{u}_i$. The displacement operator $\boldsymbol{u}_i = \lambda_{\rm ho}(\boldsymbol{a}_i^{\dagger} + \boldsymbol{a}_i)$ from the trap's origin $\boldsymbol{R}_i^{(0)}$ measures distance in units of the oscillator length $\lambda_{\rm ho} = \sqrt{\hbar/(2m\omega_{\rm ho})}$. The vector joining the two atoms will be denoted $\boldsymbol{R}_{21} = \boldsymbol{R}_2 - \boldsymbol{R}_1$, with a distance $\boldsymbol{R}_0 = \boldsymbol{R}_2^{(0)} - \boldsymbol{R}_1^{(0)}$ between the traps such that $k_{\rm in}R_0 \gg 1$.

Transition operator for photon double scattering. – Since the atoms are well separated, the total double-scattering process is described by the product of two individual scattering

processes with free propagation in between. In the far-field photon propagator $e^{ik'|\boldsymbol{R}_{21}|}/|\boldsymbol{R}_{21}|$, we expand the absolute distance to linear order in the small relative displacement: $|\boldsymbol{R}_{21}| \approx R_0 + \hat{\boldsymbol{R}}_0\,(\boldsymbol{u}_2 - \boldsymbol{u}_1)$. In the exponential, the zeroth-order contribution R_0 drops out from all interference quantities, whereas the linear term generates a phase difference between the amplitudes of A and B and must be kept. To this leading order, the denominator can be taken constant.

The transition operator [8] for way A takes a useful form in time representation and interaction picture, where $\boldsymbol{R}_j(t) = e^{iH_0t/\hbar}\boldsymbol{R}_j e^{-iH_0t/\hbar}$. Up to irrelevant prefactors,

$$T_{\mathrm{A}} = \int_0^\infty \int_0^\infty e^{i\gamma(s+t)} e^{-i\boldsymbol{k}_{\mathrm{out}}\boldsymbol{R}_2(0)} e^{i\boldsymbol{k}'\boldsymbol{R}_2(-s)} e^{-i\boldsymbol{k}'\boldsymbol{R}_1(-s)} e^{i\boldsymbol{k}_{\mathrm{in}}\boldsymbol{R}_1(-s-t)}\,\mathrm{d}t\,\mathrm{d}s. \tag{2}$$

Read from right to left, it describes how the photon is scattered first by atom 1, then propagates with $\boldsymbol{k}' \equiv k_{\mathrm{in}}\hat{\boldsymbol{R}}_0$, and is finally scattered by atom 2. The transition operator T_{B} of the reverse way B is obtained from T_{A} by the substitution $\boldsymbol{R}_1 \leftrightarrow \boldsymbol{R}_2, \boldsymbol{k}' \to -\boldsymbol{k}'$. The complex detuning $\gamma = \omega_{\mathrm{in}} - \omega_0 + i\Gamma/2$ of the probe frequency ω_{in} from the transition frequency includes the spontaneous decay rate or inverse lifetime Γ. Retardation times of order R_0/c have been neglected in the time arguments of the operators. Indeed, the photon scattering by two atoms defines two distinct time scales for the atomic motion: first, the total inverse detuning $|\gamma|^{-1}$ which is simply $2/\Gamma$ at resonance. Second, the free propagation time R_0/c from one atom to the other. For resonant atomic scatterers with $1/\Gamma$ of order 10^7 s or larger, and typical distances R_0 of $1\,\mathrm{mm}$ or less, one has $R_0/c \ll 1/\Gamma$ such that the scattering is resonance-dominated, and the free propagation time can be safely neglected. In all of the following, we have in mind the limit of quasi-free atoms and therefore consider the case of shallow traps $\omega_{\mathrm{ho}} \ll \Gamma$ in which the oscillation period of an atom is much larger than the time Γ^{-1} it takes to scatter a photon.

Quantitative quantum duality. – The incident photon can choose between two *a priori* indistinguishable paths, way A or B. Following Englert's fashionable choice [3], we call this binary degree of freedom a qubit. Let $|\mathrm{A}\rangle$ and $|\mathrm{B}\rangle$ denote the choice of way A and B. Scattering entangles the qubit with the motional degrees of freedom. Consequently, the atomic oscillator states can serve as a which-path detector.

Prior to scattering, the total initial state of qubit and detector is $\rho_{\mathrm{tot}}^{(\mathrm{i})} = \rho_{\mathrm{AB}}^{(\mathrm{i})} \otimes \rho^{(\mathrm{i})}$. The qubit is in the pure state $\rho_{\mathrm{AB}}^{(\mathrm{i})} = \frac{1}{2}\left(|\mathrm{A}\rangle + |\mathrm{B}\rangle\right)\left(\langle\mathrm{A}| + \langle\mathrm{B}|\right)$, the symmetric superposition of equally probable ways. An external cooling laser field serves as a thermal bath for the trapped atoms. The detector is thus in a thermal state, $\rho^{(\mathrm{i})} = e^{-\beta H_{\mathrm{ext}}}/Z$ at inverse temperature $\beta = 1/k_{\mathrm{B}}T$ with partition function $Z = \mathrm{tr}\{e^{-\beta H_{\mathrm{ext}}}\}$. The total final state is then obtained by applying the transition operators associated with way A and B:

$$\rho_{\mathrm{tot}}^{(\mathrm{f})} = \frac{T\rho_{\mathrm{tot}}^{(\mathrm{i})}T^\dagger}{I_{\mathrm{A}} + I_{\mathrm{B}}}, \qquad T = |A\rangle\langle A| \otimes T_{\mathrm{A}} + |B\rangle\langle B| \otimes T_{\mathrm{B}}. \tag{3}$$

In general, T is not a unitary operator since it describes only the scattering amplitude around the backscattering direction. The factor $I_{\mathrm{A}} + I_{\mathrm{B}} \equiv \mathrm{tr}\{T_{\mathrm{A}}\rho^{(\mathrm{i})}T_{\mathrm{A}}^\dagger\} + \mathrm{tr}\{T_{\mathrm{B}}\rho^{(\mathrm{i})}T_{\mathrm{B}}^\dagger\}$ guarantees, however, that $\rho_{\mathrm{tot}}^{(\mathrm{f})}$ is properly normalized.

Adapting Englert's general definitions [3], we can express the *visibility* $\mathcal{V}$ and the *distinguishability* $\mathcal{D}$ obeying the fundamental *duality relation* $\mathcal{V}^2 + \mathcal{D}^2 \le 1$ as follows:

$$\mathcal{V} = 2\frac{\left|\mathrm{tr}\{T_{\mathrm{B}}\rho^{(\mathrm{i})}T_{\mathrm{A}}^\dagger\}\right|}{I_{\mathrm{A}} + I_{\mathrm{B}}}, \qquad \mathcal{D} = \frac{\mathrm{tr}\left|T_{\mathrm{A}}\rho^{(\mathrm{i})}T_{\mathrm{A}}^\dagger - T_{\mathrm{B}}\rho^{(\mathrm{i})}T_{\mathrm{B}}^\dagger\right|}{I_{\mathrm{A}} + I_{\mathrm{B}}}. \tag{4}$$

The visibility is the ratio of interference contribution and background intensity $I_A + I_B$ and therefore equal to the CBS contrast. The distinguishability $\mathcal{D}$ describes the maximum which-way information available in principle, *i.e.*, that can be extracted by the additional measurement of an optimal detector observable; $\mathrm{tr}\,|X| = \mathrm{tr}\{\sqrt{X^\dagger X}\}$ denotes the trace-class norm of the operator X. Another interesting quantity is the *predictability*

$$\mathcal{P} = \frac{\left| \mathrm{tr}\left\{ T_A \rho^{(i)} T_A^\dagger - T_B \rho^{(i)} T_B^\dagger \right\} \right|}{I_A + I_B} = \frac{|I_A - I_B|}{I_A + I_B}. \tag{5}$$

The predictability measures the amount of which-way information available *a priori*, as for instance in unbalanced interferometers like Young's double slits with different widths. For balanced interferometers $I_A = I_B$, the predictability vanishes, $\mathcal{P} = 0$.

With the help of these quantities, we can quantify the breakdown of coherent photon backscattering by mobile atoms, both at zero and finite temperature. The distinguishability $\mathcal{D}$ is difficult to evaluate in our case because it is defined via the trace-class norm of operators on the infinite-dimensional Hilbert space of harmonic oscillators. On the contrary, visibility $\mathcal{V}$ and predictability $\mathcal{P}$ involve a thermal harmonic average of products like $T_A^\dagger T_B$ or $T_A^\dagger T_A$. Since the transition operator (2) contains only exponentials linear in the displacement, taking the trace amounts to Gaussian integration.

Zero temperature. – At zero temperature, the atoms are initially in their respective harmonic oscillator ground states. In other words, the detectors are prepared in pure states. Setting $\rho^{(i)} = |\Psi\rangle\langle\Psi|$ in (4) permits to show that in this case the duality relation is always saturated: $\mathcal{D}^2 = 1 - \mathcal{V}^2$. Thus, we can use the visibility $\mathcal{V}$, much easier to calculate than the distinguishability $\mathcal{D}$, in order to understand how much which-way information is present and how it is encoded. Carrying out the thermal average, we find in the limit $T \to 0$

$$\mathcal{D}^2 = \varrho^2(\omega_{in}) + 2\zeta^2(\omega_{in}) = 1 - \mathcal{V}^2, \tag{6}$$
$$\mathcal{P}^2 = \varrho^2(\omega_{in}), \tag{7}$$

neglecting higher-order terms $O(\zeta^4, \chi^3, \zeta^2\chi)$ in the two relevant small parameters defined as follows: The influence of atomic recoil without harmonic trapping is encoded in the factor

$$\varrho(\omega_{in}) = 4(\hat{\boldsymbol{R}}_0 \cdot \hat{\boldsymbol{k}}_{in})\frac{\delta}{|\gamma|}\chi, \qquad \chi \equiv \frac{\omega_{rec}}{|\gamma|}, \tag{8}$$

with $\hbar\omega_{rec} = \hbar^2 k_{in}^2/(2m)$ the recoil energy, $\delta = \omega_{in} - \omega_0$ the probe detuning from the atomic resonance, and $\gamma = \delta + i\Gamma/2$ as before. The harmonic trap enters through the parameter

$$\zeta^2(\omega_{in}) \equiv \frac{4\omega_{rec}\omega_{ho}}{|\gamma|^2}. \tag{9}$$

It is worthwhile to discuss the physical significance of both parameters.

Let us first consider a very shallow trap $\omega_{ho} \ll \omega_{rec}$, where the free recoil effect described by $\varrho(\omega_{in})$ dominates. At least to order χ^2, we find $\mathcal{P}^2 + \mathcal{V}^2 = 1$. Together with the general property $\mathcal{P} \leq \mathcal{D}$ (read: the *a priori* information cannot be larger than the total available information) this implies that all which-way information is actually available *a priori*: $\mathcal{D} = \mathcal{P}$. Just as in the case of other asymmetric interferometers, this predictability is due to unbalanced scattering amplitudes. At perpendicular scattering $\hat{\boldsymbol{R}}_0 \cdot \hat{\boldsymbol{k}}_{in} = 0$, the situation is completely symmetric, and both paths are equally probable. But for the extreme case $\hat{\boldsymbol{R}}_0 \cdot \hat{\boldsymbol{k}}_{in} = \pm 1$ of atoms in line with the probe, the atom in front scatters either the incident probe photon at the
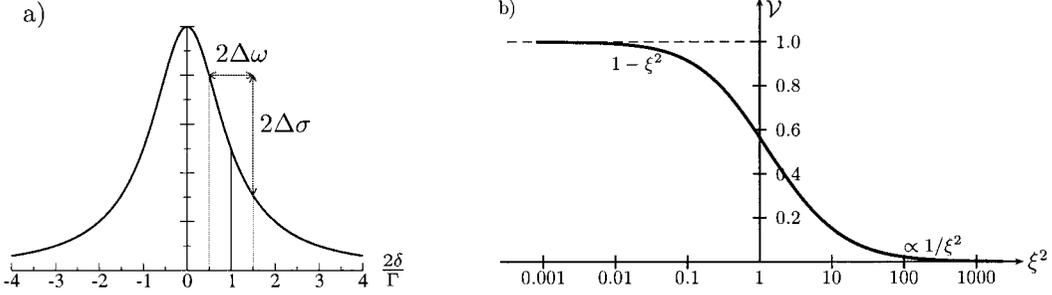
a)

b)

Fig. 2 – (a) Frequency-shift mechanism: a small frequency change $\Delta\omega$ on the steep slope of a resonance curve implies a large scattering cross-section difference $\Delta\sigma$ that can provide which-path information. (b) CBS visibility (4) as a function of the thermal Doppler shift (10) at resonance $\delta = 0$.

laser frequency or the already scattered photon on its way out again, now at laser frequency minus twice the recoil. A different frequency generally implies a different scattering cross-section such that the two paths have different probabilities. For a given position configuration, this information is known *a priori* without the necessity to perform any measurement. More quantitatively, the relative change in the resonant cross-section $\sigma(\delta) = \sigma_0[1+(2\delta/\Gamma)^2]^{-1}$ under a small frequency change $\Delta\omega \ll \Gamma$ then is $|\Delta\sigma/\sigma| = \Delta\omega\, 2\delta/|\gamma|^2$, see fig. 2(a). For the actual frequency change $\Delta\omega = 2(\hat{\boldsymbol{R}}_0 \cdot \hat{\boldsymbol{k}}_{\rm in})\omega_{\rm rec}$, one finds exactly $|\Delta\sigma/\sigma| = \varrho(\omega_{\rm in})$. To this order, the predictability vanishes at exact detuning, since a small frequency change on the flat top of the resonance Lorentzian has no effect.

Let us now interpret the influence of harmonic trapping at zero temperature. The predictability (7) contains no contribution in $\zeta^2(\omega_{\rm in})$, which indicates that this parameter encodes which-path information that may only be revealed *a posteriori* by an appropriate measurement on the detector. In a temporal picture (see fig. 1(b)), one can determine which way the photon has taken if one can measure which atom has scattered the photon first. This is only possible if the initial position uncertainty λ is smaller than the distance Δx travelled by the first scattering atom. More quantitatively, the first scatterer takes up the recoil in the direction of $\hat{\boldsymbol{k}}_{\rm in}$ and travels the distance $\Delta x = v_{\rm rec}\Delta t = \sqrt{2\hbar\omega_{\rm rec}/m}\Delta t$ during the whole time $\Delta t = 2|\gamma|^{-1}$ until the emission of the final photon by the second atom[1]. At that moment, the second atom receives the same recoil in the direction $\hat{\boldsymbol{k}}_{\rm in}$, such that the final momentum states for both ways become indistinguishable (note that the recoil in the direction joining the atoms is exchanged instantaneously, because we can neglect the propagation time). The zero-temperature position uncertainty is the oscillator length $\lambda_{\rm ho}$, which finally shows that which-path information is indeed present on the scale $\Delta x/\lambda_{\rm ho} = 2\zeta(\omega_{\rm in})$.

Finite temperatures. – If the atoms are coupled to a thermal bath, the detector is not prepared in a pure state, and one expects that the duality relation no longer saturates. We thus have to calculate visibility and distinguishability separately.

After the Gaussian thermal average, the visibility is given in terms of a product of amplitudes like (2) for way A and B. It can be evaluated numerically for all values of the detuning δ and of the thermal trap parameter $\xi^2(T, \omega_{\rm in}) \equiv \coth(\frac{1}{2}\beta\hbar\omega_{\rm ho})\zeta^2(\omega_{\rm in})$. This param-

---

[1]Δt is simply twice the total contribution per atom that can be justified by a stationary-phase argument if both the Wigner time delay of the phase and the amplitude change of the resonant scattering function $t(\delta) = t_0/(\delta + i\Gamma/2)$ are taken into account.

eter corresponds to the zero-temperature trap parameter (9) at effective oscillator excitation $[2n(T)+1]\omega_{\mathrm{ho}}$, with $n(T) = [e^{\beta\hbar\omega_{\mathrm{ho}}} - 1]^{-1}$ the Bose-Einstein distribution function. For highly excited atoms $\beta\hbar\omega_{\mathrm{ho}} \ll 1$ the oscillator frequency drops out of the definition of the thermal trap parameter which becomes

$$\xi_{\mathrm{cl}} = \frac{2k_{\mathrm{in}}v_{\mathrm{rms}}}{|\gamma|}, \tag{10}$$

the classical thermal Doppler shift $(v_{\mathrm{rms}}^2 = \langle \boldsymbol{v}^2 \rangle)$ in units of the linewidth. Figure 2(b) shows a plot of the visibility at exact resonance $\delta = 0$ as a function of ξ_{cl}^2 on a semi-logarithmic scale. Here, we concentrate on the effect of harmonic trapping at finite temperature and neglect the free recoil contribution, which is justified if $\chi/\xi_{\mathrm{cl}}^2 \ll 1$. The visibility decreases monotonically with temperature. For $\xi_{\mathrm{cl}}^2 \ll 1$, we can expand to lowest order and find analytically (compare with eq. (6)):

$$\mathcal{V}^2 = 1 - 2\xi_{\mathrm{cl}}^2, \tag{11}$$

At this point, we recover the case of free thermal atoms. Our result (11) agrees with the CBS contrast calculated in [9] for the low-temperature case $\xi_{\mathrm{cl}} \ll 1$ (if the avarage medium effect included there is disregarded). Note that our calculation is also valid in the high-temperature regime $\xi_{\mathrm{cl}} \gg 1$, where the visibility goes to zero as $\mathcal{V} \sim \xi^{-2}$.

It is too difficult to evaluate analytically the trace-class norm for the distinguishability $\mathcal{D}$ with the transition operators in the general form (2). Therefore, we formally expand the exponentials in powers of the Lamb-Dicke parameter $k_{\mathrm{in}}\lambda_{\mathrm{ho}} = \sqrt{\omega_{\mathrm{rec}}/\omega_{\mathrm{ho}}}$. The leading-order expression for $\mathcal{D}$ in the shallow-trap limit $\beta\hbar\omega_{\mathrm{ho}} \ll 1$ becomes

$$\mathcal{D} = \frac{2\sqrt{2}|\delta|}{|\gamma|^2} v_{\mathrm{rec}}\beta\omega_{\mathrm{ho}} \mathrm{tr} \left| e^{-\beta\hbar\omega_{\mathrm{ho}}\hat{a}^{\dagger}\hat{a}} \hat{p} \right|. \tag{12}$$

Here, $\hat{p} = (\hbar/2i\lambda_{\mathrm{ho}})[\hat{a} - \hat{a}^{\dagger}]$ is the momentum associated with $\hat{a} = (\boldsymbol{a}_1 - \boldsymbol{a}_2) \cdot \hat{\boldsymbol{k}}_{\mathrm{in}}$, the anti-symmetric oscillator mode projected onto the probe direction $\hat{\boldsymbol{k}}_{\mathrm{in}}$. It is reasonable that only this mode should be relevant: symmetric motion cannot encode differential information about the paths (and could be disposed of by transformation into a co-moving frame), whereas the momentum along the only other available direction $\hat{\boldsymbol{R}}_0$ is exchanged instantaneously. To linear order in $\bar{\beta} = \beta\hbar\omega_{\mathrm{ho}}$, the positive operator $V = e^{-\bar{\beta}\hat{a}^{\dagger}\hat{a}}$ can be moved outside the absolute value (using $|V\hat{p}| \approx \sqrt{(V^{\dagger}V)(\hat{p}^{\dagger}\hat{p})} \approx |V||\hat{p}|$ since $\hat{p}$ and V commute to zeroth order in $\bar{\beta}$). This makes the distinguishability proportional to the thermal expectation value of the momentum modulus, which is easy to evaluate for a Maxwell-Boltzmann distribution:

$$\mathcal{D} = \sqrt{2}\frac{|\delta|}{|\gamma|}\frac{2v_{\mathrm{rec}}}{|\gamma|}\frac{\langle|\hat{p}|\rangle}{\hbar} = \frac{2}{\sqrt{\pi}}\frac{|\delta|}{|\gamma|}\xi_{\mathrm{cl}}. \tag{13}$$

It can be read as the relative shift $\mathcal{D} = \pi^{-1/2}|\Delta\sigma/\sigma|$ of the resonant cross-section under the small Doppler frequency change $\Delta\omega = 2k_{\mathrm{in}}v_{\mathrm{rms}}$. This suggests an interpretation in terms of the frequency-shift mechanism (cf. fig. 2(a)): By measuring the actual atomic velocities *a posteriori*, one can determine which way was more probable: if atom 1 is found to move towards a red-detuned probe and atom 2 to move away from it, way A was more probable than B because atom 1 had a larger absorption cross-section —this is simply the principle of Doppler cooling. Contrary to the zero-temperature case, now the predictability $\mathcal{P}$ is zero to the order considered because the interferometer symmetry is re-established by the isotropic thermal average. Naturally, as in the case of harmonic trapping at zero temperature, the distinguishability vanishes also at exact resonance $\delta = 0$ because no differential information

can be encoded on the flat top of the resonance Lorentzian. In the high-temperature limit $\xi_{\mathrm{cl}} \gg 1$, *i.e.*, for very large frequence shifts $\Delta\omega \gg \Gamma$ with moderate detuning, one explores the flat wings of the Lorentzian, and the distinguishability must then drop to zero.

At finite temperature, the quantum duality is no longer saturated and reads to order ξ_{cl}^2:

$$\mathcal{V}^2 + \mathcal{D}^2 = 1 - 2 \left[1 - \frac{2}{\pi} \frac{\delta^2}{|\gamma|^2} \right] \left(\frac{2 k_{\mathrm{in}} v_{\mathrm{rms}}}{|\gamma|} \right)^2 . \tag{14}$$

This value of $\mathcal{D}$ assumes an optimal measurement within just the system of center-of-mass motion. But the initial thermal distribution requires the presence of a thermal bath (*e.g.*, cooling lasers). The total system (detector plus bath) is described by a pure state for which the quantum duality saturates. However, the which-path information could only be retrieved by an optimal measurement including all bath degrees of freedom that are not under our control. Measurements on just the detector necessarily correspond to a non-optimal measurement with respect to the whole system and therefore imply a reduced distinguishability.

Summary. –   We have studied the coherent backscattering effect from two trapped atoms, an interesting case study for quantitative quantum duality in a physically realistic setting. At zero temperature, inelastic scattering due to the recoil effect provides *a priori* which-path information. Further which-path information can be gained by measuring which atom has scattered the photon first, which is possible if the initial position uncertainty is small enough. At finite temperature, the atomic movement destroys the interference once the average Doppler shift becomes of the order of the resonance width. Which-path information resides in the atomic velocities via resonance conditions, but the duality inequality no longer saturates because an initial thermal state corresponds to a non-optimal detector preparation.

We hope that these considerations may stimulate further work, both experimental and theoretical, on the interference probing of quantum dynamics of trapped particles.

$$* * *$$

REFERENCES

[1] EICHMANN U. *et al.*, *Phys. Rev. Lett.*, **70** (1993) 2359; ITANO W. M. *et al.*, *Phys. Rev. A*, **57** (1998) 4176.
[2] ESCHNER J., *Eur. Phys. J. D*, **22** (2003) 341.
[3] ENGLERT B.-G., *Phys. Rev. Lett.*, **77** (1996) 2154; ENGLERT B.-G. and BERGOU J. A., *Opt. Commun.*, **179** (2000) 337.
[4] DÜRR S., NONN T. and REMPE G., *Phys. Rev. Lett.*, **81** (1998) 5705.
[5] VAN ALBADA M. P. and LAGENDIJK A., *Phys. Rev. Lett.*, **55** (1985) 2692; WOLF P. E. and MARET G., *Phys. Rev. Lett.*, **55** (1985) 2696.
[6] LABEYRIE G. *et al.*, *J. Opt. B*, **2** (2000) 672.
[7] LOUDON R., *The Quantum Theory of Light* (Oxford University Press) 2003.
[8] COHEN-TANNOUDJI C., DUPONT-ROC J. and GRYNBERG G., *Atom-Photon Interactions* (Wiley) 1992.
[9] WILKOWSKI D. *et al.*, *Physica B*, **328** (2003) 157.