

Burkard group
Department of physics
University of Konstanz

Optical absorption of a single layer of graphene

Sven Deuschländer

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Supervised by

Eleftheria Kavousanaki

I. INTRODUCTION

The purpose of this article is to describe the interaction of an electromagnetic field with graphene and is mainly motivated by reference [1]. We will consider the case of an electromagnetic field propagating perpendicular to a single graphene layer and will discuss the transmissivity and reflectivity of the graphene sheet. We will show that there exists a 2.2% absorption of the incident field which leads to a non-zero conductivity on the graphene layer. Moreover we will show that this optical absorption is frequency independent and depends only on the fine structure constant, which is an impressive macroscopic observation of nature's physical constants. Additionally, the theoretical prediction of the transmissivity of a bilayer graphene will be briefly commented [2]. Finally we will compare with experimental results for the visual transparency of graphene [3].

II. ELECTROMAGNETIC SCATTERING PROBLEM

We start with the scattering geometry given in figure 1. A single layer of graphene of the size A_s is placed at the interface of two media 1 and 2 with the electrical permittivities ϵ_1 and ϵ_2 . The graphene layer, lying in the x, y -plane, is irradiated with an electromagnetic wave, polarised parallel to the interface, and the plane of incidence is the x, z -plane. The wavevector \mathbf{k} , the amplitudes of the incident and reflected electric field \mathbf{E}_I and \mathbf{E}_R with the corresponding angle θ_1 to the interface normal vector \mathbf{n} , and the transmitted field \mathbf{E}_T with θ_2 , respectively read

$$\mathbf{k}_i = \begin{pmatrix} k_i^x \\ 0 \\ k_i^z \end{pmatrix} \quad \text{and} \quad \mathbf{E}_i = \begin{pmatrix} E_i^x \\ 0 \\ E_i^z \end{pmatrix} \quad \text{with } i = I, R, T. \quad (1)$$

With the graphene charge density ρ , the boundary conditions at the interface for the electric displacement field \mathbf{D} and the electric field \mathbf{E} hold

$$\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \rho, \quad (2)$$

$$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0, \quad (3)$$

and can be written out as

$$\epsilon_1 \epsilon_0 (E_I + E_R) \sin \theta_1 - \epsilon_2 \epsilon_0 E_T \sin \theta_2 = \rho, \quad (4)$$

$$(E_I - E_R) \cos \theta_1 - E_T \cos \theta_2 = 0, \quad (5)$$

where ϵ_0 is the vacuum permittivity. The interaction of the electromagnetic field with the charge carriers of the graphene layer produces a two-dimensional current density $\mathbf{j}(\omega)$ of electrons, given by Ohm's law

$$\mathbf{j}(\omega) = \sigma(\omega) \mathbf{E}(\omega), \quad (6)$$

where the complex tensor $\sigma(\omega)$ is the optical conductivity, reading

$$\sigma(\omega) = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}. \quad (7)$$

The current density is connected to the charge density via the continuity equation

$$\nabla \cdot \mathbf{j}(\mathbf{r}, t) = -\frac{\partial}{\partial t} \rho(\mathbf{r}, t). \quad (8)$$

By Fourier transformation of equation (8) and concerning the scattering geometry one observes

$$\rho(\omega) = \frac{1}{\omega} \mathbf{j}(\omega) \cdot \mathbf{k} = \frac{1}{\omega} j_x(\omega) k_x = \frac{k_x}{\omega} \sigma_{xx}(\omega) E_T \cos \theta_2. \quad (9)$$

Combining equations (4), (5) and (9) with the expression for the transmissivity T and reflectivity R

$$T = \frac{k_T^z}{k_I^z} \left| \frac{E_T}{E_I} \right|^2 = \sqrt{\frac{\epsilon_2 \cos \theta_2}{\epsilon_1 \cos \theta_1}} \left| \frac{E_T}{E_I} \right|^2 \quad \text{and} \quad R = \left| \frac{E_R}{E_I} \right|^2,$$

one gets for normal incidence ($\theta_1 = 0$)

$$T = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{4(\epsilon_1 \epsilon_0)^2}{\left| (\sqrt{\epsilon_1 \epsilon_2} + \epsilon_1) \epsilon_0 + \sqrt{\epsilon_1} \frac{\sigma_{xx}(\omega)}{c} \right|^2}, \quad (10)$$

$$R = \frac{\left| (\sqrt{\epsilon_1 \epsilon_2} - \epsilon_1) \epsilon_0 + \sqrt{\epsilon_1} \frac{\sigma_{xx}(\omega)}{c} \right|^2}{\left| (\sqrt{\epsilon_1 \epsilon_2} + \epsilon_1) \epsilon_0 + \sqrt{\epsilon_1} \frac{\sigma_{xx}(\omega)}{c} \right|^2}. \quad (11)$$

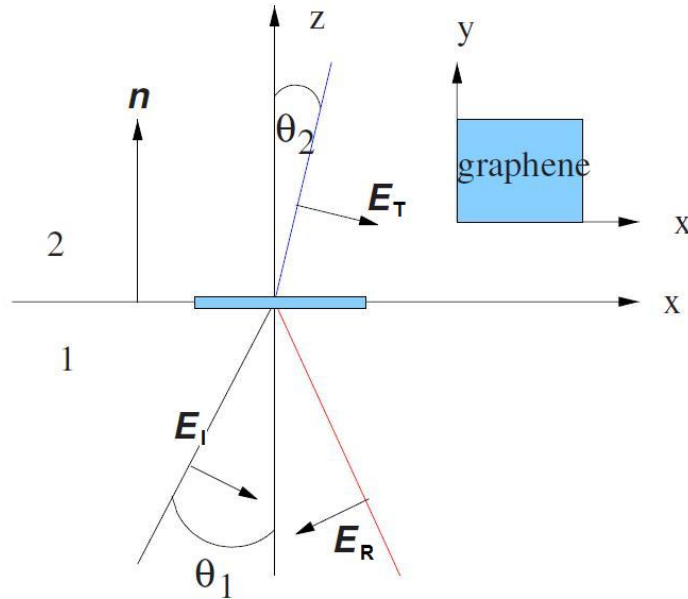


Figure 1: Electromagnetic scattering geometry [1].

The appearance of the optical conductivity in the transmissivity and reflectivity reflects a loss of energy flux due to a current in the graphene layer. The result is the absorption of energy and the condition $T + R = 1$ does not longer hold. The absorption A reads

$$A = 1 - T - R = \frac{4\epsilon_1\sqrt{\epsilon_1}\epsilon_0\Re\sigma_{xx}(\omega)}{c\left|\left(\sqrt{\epsilon_1\epsilon_2} + \epsilon_1\right)\epsilon_0 + \sqrt{\epsilon_1}\frac{\sigma_{xx}(\omega)}{c}\right|^2}. \quad (12)$$

III. HAMILTONIAN AND CURRENT DENSITY OPERATOR

To get an expression for the optical conductivity, one first has to calculate the current density operator j_x , which can be obtained from the Hamiltonian of the system. In the tight-binding theory for nearest neighbors the Hamiltonian is given in second quantisation by

$$H = -t \sum_{\mathbf{r},s} \sum_{\boldsymbol{\delta}=\boldsymbol{\delta}_1,\boldsymbol{\delta}_2,\boldsymbol{\delta}_3} \left[a_s^\dagger(\mathbf{r}) b_s(\mathbf{r} + \boldsymbol{\delta}) + b_s^\dagger(\mathbf{r} + \boldsymbol{\delta}) a_s(\mathbf{r}) \right], \quad (13)$$

where t is the nearest neighbor hopping parameter. $a_s^\dagger(\mathbf{r})$ [$a_s(\mathbf{r})$] and $b_s^\dagger(\mathbf{r})$ [$b_s(\mathbf{r})$] are the creation [annihilation] operators for an electron with spin s at site \mathbf{r} of sublattice A and B respectively, and $\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \boldsymbol{\delta}_3$ are the vectors connecting nearest neighbors. The lattice structure is given in figure 2. With the carbon-carbon distance a , the nearest neighbor connecting vectors read

$$\boldsymbol{\delta}_1 = \frac{a}{2} \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}, \quad \boldsymbol{\delta}_2 = \frac{a}{2} \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\delta}_3 = -a \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (14)$$

The current density operator can now be obtained from the Hamiltonian mechanics with

$$j_x = -\frac{\partial H}{\partial A_x}, \quad (15)$$

where A_x is the x -component of the vector potential. To find the effect of the vector potential on the Hamiltonian, it is easier to make a Fourier transformation to the momentum space. This has to be done by transforming the creation and annihilation operators, reading

$$a_s^\dagger(\mathbf{r}) = \frac{1}{\sqrt{A_s}} \sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}} a_{\mathbf{k},s}^\dagger \quad \text{and} \quad b_s^\dagger(\mathbf{r}) = \frac{1}{\sqrt{A_s}} \sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}} b_{\mathbf{k},s}^\dagger, \quad (16)$$

where $a_{\mathbf{k},s}^\dagger$ and $b_{\mathbf{k},s}^\dagger$ are the creation operators in momentum space. The annihilation operators can be obtained similarly by complex conjugating the above expressions. The Hamiltonian in momentum space now reads

$$\begin{aligned} H &= -\frac{t}{A_s} \sum_{\mathbf{r},s,\boldsymbol{\delta}} \left[\sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}} a_{\mathbf{k},s}^\dagger \sum_{\mathbf{k}'} e^{i\mathbf{k}'\cdot(\mathbf{r}+\boldsymbol{\delta})} b_{\mathbf{k}',s} + \sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot(\mathbf{r}+\boldsymbol{\delta})} b_{\mathbf{k},s}^\dagger \sum_{\mathbf{k}'} e^{i\mathbf{k}'\cdot\mathbf{r}} a_{\mathbf{k}',s} \right] \\ &= -\frac{t}{A_s} \sum_{s,\boldsymbol{\delta},\mathbf{k},\mathbf{k}'} \left[e^{i\mathbf{k}'\cdot\boldsymbol{\delta}} a_{\mathbf{k},s}^\dagger b_{\mathbf{k}',s} \sum_{\mathbf{r}} e^{i(\mathbf{k}'-\mathbf{k})\mathbf{r}} + e^{-i\mathbf{k}\cdot\boldsymbol{\delta}} b_{\mathbf{k},s}^\dagger a_{\mathbf{k}',s} \sum_{\mathbf{r}} e^{i(\mathbf{k}'-\mathbf{k})\mathbf{r}} \right] \\ &= -t \sum_{\mathbf{k},s,\boldsymbol{\delta}} \left[e^{i\mathbf{k}\cdot\boldsymbol{\delta}} a_{\mathbf{k},s}^\dagger b_{\mathbf{k},s} + e^{-i\mathbf{k}\cdot\boldsymbol{\delta}} b_{\mathbf{k},s}^\dagger a_{\mathbf{k},s} \right], \end{aligned} \quad (17)$$

where we have used the identity

$$\frac{1}{A_s} \sum_{\mathbf{r}} e^{i(\mathbf{k}'-\mathbf{k})\mathbf{r}} = \delta(\mathbf{k}-\mathbf{k}'). \quad (18)$$

In presence of an electromagnetic field one has to make the substitution

$$\mathbf{p} \rightarrow \mathbf{p} + e\mathbf{A} \quad \text{or} \quad \mathbf{k} \rightarrow \mathbf{k} + \frac{e}{\hbar}\mathbf{A}, \quad (19)$$

where \mathbf{A} is the vector potential, e the elementary charge and \hbar the Planck constant, leading to the Hamiltonian in momentum space in presence of an electromagnetic field

$$H = -t \sum_{\mathbf{k},s,\boldsymbol{\delta}} \left[e^{i(\mathbf{k}+\frac{e}{\hbar}\mathbf{A})\cdot\boldsymbol{\delta}} a_{\mathbf{k},s}^\dagger b_{\mathbf{k},s} + e^{-i(\mathbf{k}+\frac{e}{\hbar}\mathbf{A})\cdot\boldsymbol{\delta}} b_{\mathbf{k},s}^\dagger a_{\mathbf{k},s} \right]. \quad (20)$$

Transformed back into real space, the Hamiltonian reads

$$H = -t \sum_{\mathbf{r}, s, \boldsymbol{\delta}} \left[e^{i \frac{e}{\hbar} \mathbf{A} \cdot \boldsymbol{\delta}} a_s^\dagger(\mathbf{r}) b_s(\mathbf{r} + \boldsymbol{\delta}) + e^{-i \frac{e}{\hbar} \mathbf{A} \cdot \boldsymbol{\delta}} b_s^\dagger(\mathbf{r} + \boldsymbol{\delta}) a_s(\mathbf{r}) \right], \quad (21)$$

where we see that the substitution in momentum space given in equation (19) is similar to the Peierls substitution in real space, given by

$$t \rightarrow t \cdot e^{i \frac{e}{\hbar} \mathbf{A} \cdot \boldsymbol{\delta}}. \quad (22)$$

The current density operator j_x can now be calculated with equation (15), reading

$$j_x = \frac{ite}{\hbar} \sum_{\mathbf{r}, s, \boldsymbol{\delta}} \left[\delta_x \cdot e^{i \frac{e}{\hbar} \mathbf{A} \cdot \boldsymbol{\delta}} a_s^\dagger(\mathbf{r}) b_s(\mathbf{r} + \boldsymbol{\delta}) - \delta_x \cdot e^{-i \frac{e}{\hbar} \mathbf{A} \cdot \boldsymbol{\delta}} b_s^\dagger(\mathbf{r} + \boldsymbol{\delta}) a_s(\mathbf{r}) \right]. \quad (23)$$

Expanding the exponential function up to first order in A_x with

$$e^{i \frac{e}{\hbar} A_x \delta_x} \simeq 1 + i \frac{e}{\hbar} A_x \delta_x, \quad (24)$$

yields

$$j_x \simeq j_x^P + A_x j_x^D, \quad (25)$$

with a term constant in A_x ,

$$j_x^P = \frac{ite}{\hbar} \sum_{\mathbf{r}, s, \boldsymbol{\delta}} \left[\delta_x a_s^\dagger(\mathbf{r}) b_s(\mathbf{r} + \boldsymbol{\delta}) - \delta_x b_s^\dagger(\mathbf{r} + \boldsymbol{\delta}) a_s(\mathbf{r}) \right], \quad (26)$$

and linear in A_x ,

$$j_x^D = -\frac{te^2}{\hbar^2} \sum_{\mathbf{r}, s, \boldsymbol{\delta}} \left[\delta_x^2 a_s^\dagger(\mathbf{r}) b_s(\mathbf{r} + \boldsymbol{\delta}) - \delta_x^2 b_s^\dagger(\mathbf{r} + \boldsymbol{\delta}) a_s(\mathbf{r}) \right]. \quad (27)$$

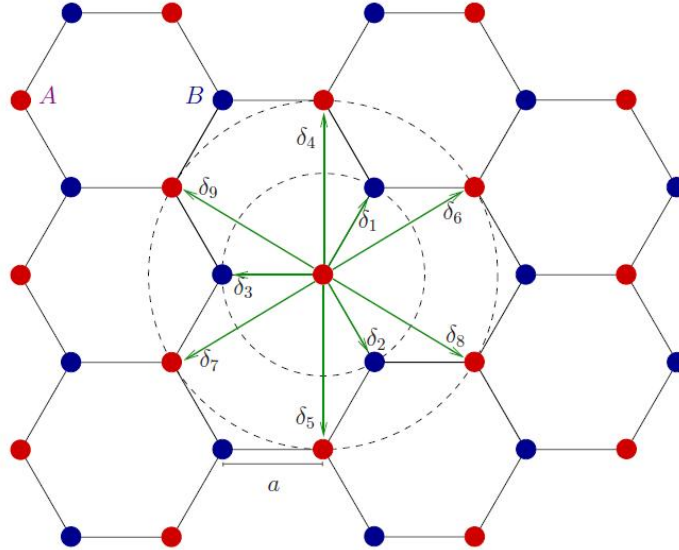


Figure 2: Graphene lattice structure [1].

IV. OPTICAL CONDUCTIVITY

An expression of the optical conductivity $\sigma_{xx}(\omega)$ can be obtained from the Kubo formalism, that treats the current \mathbf{j} as a linear response to an electromagnetic perturbation with the response function

$$\sigma_{xx}(\omega) = \frac{\langle j_x^D \rangle}{iA_s(\omega + i\varepsilon)} + \frac{\Lambda_{xx}(\omega + i\varepsilon)}{i\hbar A_s(\omega + i\varepsilon)}, \quad (28)$$

where $\varepsilon \rightarrow 0^+$. The function $\Lambda_{xx}(\omega + i\varepsilon)$ is given by

$$\Lambda_{xx}(\omega + i\varepsilon) = \int_0^{\hbar\beta} d\tau e^{(\omega+i\varepsilon)\tau} \langle j_x^P(\tau) j_x^P(0) \rangle, \quad (29)$$

where $\beta = (k_B T)^{-1}$, with the Boltzmann constant k_B and the temperature T , $\langle \dots \rangle$ represents the mean value in the grand canonical ensemble. $j_x^P(\tau)$ is a time dependent operator in the Heisenberg picture, reading

$$j_x^P(\tau) = e^{i(H-\mu N)\tau} j_x^P e^{-i(H-\mu N)\tau}, \quad (30)$$

with the chemical potential μ and the number operator N given by

$$N = \sum_{\mathbf{r},s} [a_s^\dagger(\mathbf{r}) a_s(\mathbf{r}) + b_s^\dagger(\mathbf{r} + \boldsymbol{\delta}_3) b_s(\mathbf{r} + \boldsymbol{\delta}_3)]. \quad (31)$$

The real and imaginary part of the conductivity $\Re\sigma_{xx}(\omega)$ and $\Im\sigma_{xx}(\omega)$ can be separated using the identity

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} \int_{-\infty}^{\infty} \frac{f(\omega)}{\omega + i\varepsilon} d\omega &= \lim_{\varepsilon \rightarrow 0} \int_{-\infty}^{-\varepsilon} \frac{f(\omega)}{\omega + i\varepsilon} d\omega + \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon}^{\infty} \frac{f(\omega)}{\omega + i\varepsilon} d\omega \\ &\quad - i\pi \int_{-\varepsilon}^{\varepsilon} \delta(\omega) f(\omega) d\omega, \end{aligned} \quad (32)$$

commonly written as

$$\frac{1}{\omega + i\varepsilon} = \text{P} \frac{1}{\omega} - i\pi \delta(\omega), \quad (33)$$

giving

$$\Re\sigma_{xx}(\omega) = - \left(\frac{\langle j_x^D \rangle}{A_s \omega} + \pi \frac{\Re\Lambda_{xx}(\omega + i\varepsilon)}{\hbar A_s} \right) \delta(\omega) + \frac{\Im\Lambda_{xx}(\omega + i\varepsilon)}{\hbar \omega A_s}, \quad (34)$$

where $\delta(\omega)$ is the Dirac delta function. It can be set to zero, since we are interested in frequencies $\omega > 0$. The imaginary part reads

$$\Im\sigma_{xx}(\omega) = - \frac{\langle j_x^D \rangle}{A_s \omega} - \frac{\Re\Lambda_{xx}(\omega + i\varepsilon)}{\hbar \omega A_s}. \quad (35)$$

We see that the real part of the conductivity is directly connected to the imaginary part of $\Lambda_{xx}(\omega + i\varepsilon)$. With the wave functions of the graphene layer in the tight-binding model

$$|\psi_{\mathbf{k}\sigma}^\pm\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta_{\mathbf{k}}/2} \\ \pm e^{i\theta_{\mathbf{k}}/2} \end{pmatrix}, \quad (36)$$

where $\theta_{\mathbf{k}} = \arctan(k_x/k_y)$, the function $\Lambda_{xx}(\omega + i\varepsilon)$ reads

$$\begin{aligned} \Lambda_{xx}(\omega + i\varepsilon) &= \int_0^{\hbar\beta} d\tau e^{(\omega+i\varepsilon)\tau} Tr \left[e^{-\beta(H-\mu N)} j_x^P(\tau) j_x^P(0) \right] \\ &= \int_0^{\hbar\beta} d\tau e^{(\omega+i\varepsilon)\tau} \left[\frac{1}{Z} \sum_{\mathbf{k}\sigma\pm} \langle \psi_{\mathbf{k}\sigma}^\pm | e^{-\beta(H-\mu N)} e^{i(H-\mu N)\tau} j_x^P e^{-i(H-\mu N)\tau} j_x^P | \psi_{\mathbf{k}\sigma}^\pm \rangle \right], \end{aligned} \quad (37)$$

where Z represents the partition function and $Tr[\dots]$ the trace in the grand canonical ensemble. With the energy bands of the graphene layer

$$E_{\mathbf{k}}^\pm = \pm t |\phi(\mathbf{k})| = \pm t \cdot \left| 1 + e^{\mathbf{k}\cdot(\boldsymbol{\delta}_1 - \boldsymbol{\delta}_3)} + e^{\mathbf{k}\cdot(\boldsymbol{\delta}_2 - \boldsymbol{\delta}_3)} \right|, \quad (38)$$

the imaginary part of $\Lambda_{xx}(\omega + i\varepsilon)$ reads

$$\begin{aligned} \Im\Lambda_{xx}(\omega + i\varepsilon) &= \frac{t^2 e^2 a^2}{8\hbar^2} \sum_{\mathbf{k}} \left[18 - 4 |\phi(\mathbf{k})|^2 + 18 \frac{[\Re\phi(\mathbf{k})]^2 - [\Im\phi(\mathbf{k})]^2}{|\phi(\mathbf{k})|^2} \right] \\ &\quad \times [n_F(-t|\phi(\mathbf{k})| - \mu) - n_F(t|\phi(\mathbf{k})| - \mu)] \\ &\quad \times \left[\pi\delta\left(\omega - \frac{2t|\phi(\mathbf{k})|}{\hbar}\right) - \pi\delta\left(\omega + \frac{2t|\phi(\mathbf{k})|}{\hbar}\right) \right], \end{aligned} \quad (39)$$

where $n_F(x)$ is the Fermi distribution and the second delta function cancels out since $\omega > 0$. In the Dirac cone approximation, where $\phi(\mathbf{k}) \propto \mathbf{k}$, one can set

$$\frac{[\Re\phi(\mathbf{k})]^2 - [\Im\phi(\mathbf{k})]^2}{|\phi(\mathbf{k})|^2} \simeq 0, \quad (40)$$

and write equation (37) only in terms of the energy bands $E_{\mathbf{k}}^{\pm}$, reading

$$\Im\Lambda_{xx}(\omega + i\varepsilon) = \frac{t^2 e^2 a^2}{8\hbar^2} \sum_{\mathbf{k}} \left[\pi\delta\left(\omega - \frac{2E_{\mathbf{k}}^+}{\hbar}\right) \right] \left[18 - 4\left(\frac{E_{\mathbf{k}}^+}{t}\right)^2 \right] \times [n_F(-E_{\mathbf{k}}^+ - \mu) - n_F(E_{\mathbf{k}}^+ - \mu)]. \quad (41)$$

The charge carriers in the graphene layer that contribute to the current are electrons which are excited from the valence to the conduction band by photons of the energy $\hbar\omega$. The absorption process of the photons is shown in figure 3. The conservation of energy and momentum leads directly to the condition

$$\hbar\omega = 2E_{\mathbf{k}}^+. \quad (42)$$

The delta function in equation (39) can be written in terms of the density of states

$$\rho(E) = \frac{1}{N_c} \sum_{\mathbf{k}} \delta(E - E_{\mathbf{k}}^+) \quad (43)$$

of the system, reading

$$\rho\left(\frac{\hbar\omega}{2}\right) = \frac{1}{N_c} \sum_{\mathbf{k}} \delta\left(\frac{\hbar\omega}{2} - E_{\mathbf{k}}^+\right) = \frac{1}{N_c} \sum_{\mathbf{k}} \frac{\hbar}{2} \delta\left(\omega - \frac{2E_{\mathbf{k}}^+}{\hbar}\right). \quad (44)$$

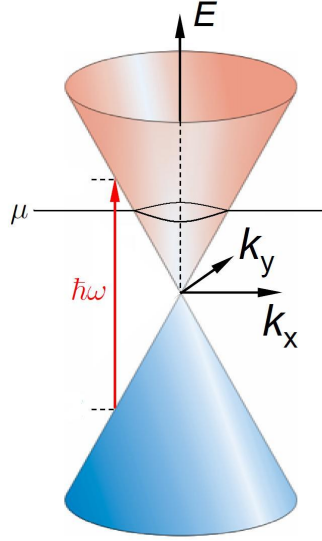


Figure 3: Excitation of an electron by absorbing a photon [3]. The momentum conservation leads to an excitation path perpendicular to the momentum plane. The Pauli principle allows only the excitation energy $\hbar\omega > 2\mu$.

With equations (39)-(41) and the sample area $A_s = N_c A_c$, where N_c is the number of atoms and A_c the area of the unit cell, the real part of the conductivity can be written as

$$\Re\sigma_{xx}(\omega) = \sigma_0 \frac{\pi t^2 a^2}{8 A_c \hbar \omega} \rho\left(\frac{\hbar\omega}{2}\right) \left[18 - \frac{(\hbar\omega)^2}{t^2} \right] \left[\tanh\frac{\hbar\omega + 2\mu}{4k_B T} + \tanh\frac{\hbar\omega - 2\mu}{4k_B T} \right], \quad (45)$$

where $\sigma_0 = (e^2/4\hbar)$. Figure 4 shows a plot of equation (45). Two things should be mentioned. First there seems to be a threshold for finite chemical potentials. This appears due to a vanishing optical conductivity below a certain photon energy. In the presence of a chemical potential and zero temperature, there are now free states below $E_{\mathbf{k}}^+ = \mu$, if $\mu > 0$, and now occupied states above $E_{\mathbf{k}}^- = \mu$, if $\mu < 0$. This leads with the Pauli principle to the condition

$$\hbar\omega > 2\mu, \quad (46)$$

which can be infringed at finite temperatures. Second and more interesting is the fact, that the real part of the optical conductivity remains nearly constant over a large frequency range, namely up to the end of the visible part of the spectrum which is about 3.1eV. This is an exceptional behavior since usually the optical conductivity in this region is dependent on the frequency. Performing an asymptotic expansion of the density of states around $E = 0$, one gets the useful result

$$\Re\sigma_{xx}(\omega) = \sigma_0 \left[\frac{1}{2} + \frac{(\hbar\omega)^2}{72t^2} \right] \left(\tanh \frac{\hbar\omega + 2\mu}{4k_B T} + \tanh \frac{\hbar\omega - 2\mu}{4k_B T} \right). \quad (47)$$

The imaginary part of the optical conductivity $\Im\sigma_{xx}(\omega)$ can be calculated similar to $\Re\sigma_{xx}(\omega)$ reading

$$\begin{aligned} \Im\sigma_{xx}(\omega) &= \sigma_0 \frac{4}{\pi\hbar\omega} \left(\mu - \frac{2\mu^3}{9t^2} \right) - \frac{\sigma_0}{\pi} \log \frac{|\hbar\omega + 2\mu|}{|\hbar\omega - 2\mu|} \\ &\quad - \frac{\sigma_0}{26\pi} \left(\frac{\hbar\omega}{t} \right)^2 \log \frac{|\hbar\omega + 2\mu|}{|\hbar\omega - 2\mu|}. \end{aligned} \quad (48)$$

For $\hbar\omega \gg \mu$, $\Im\sigma_{xx}(\omega) \rightarrow 0$, i.e. only the real part of the conductivity gives a significant contribution. Thus, in the visible region, for zero chemical potential and low temperatures we can make the approximation

$$\sigma_{xx}(\omega) \simeq \sigma_0 = \frac{e^2}{4\hbar}. \quad (49)$$

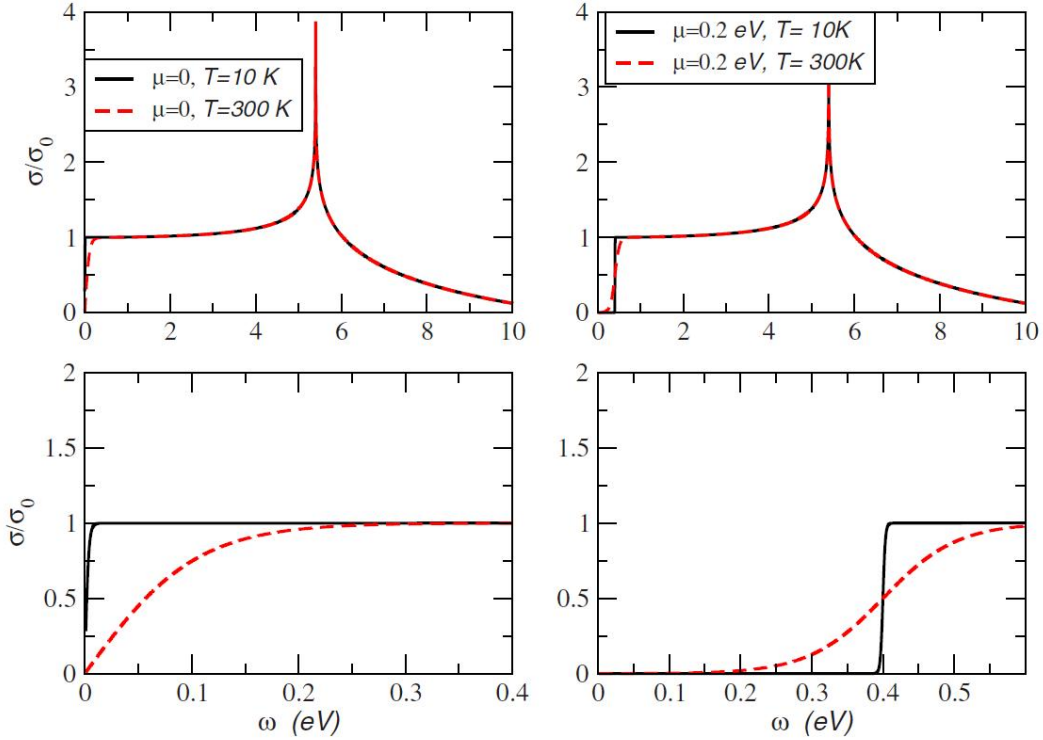


Figure 4: Real part of the optical conductivity divided by σ_0 for different chemical potentials and temperatures, using $t=2.7\text{eV}$ [1]. The lower plots are magnifications.

V. OPTICAL ABSORPTION

With the expression of the optical conductivity, the transmissivity and reflectivity for normal incidence

$$T = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{4(\epsilon_1\epsilon_0)^2}{\left| (\sqrt{\epsilon_1\epsilon_2} + \epsilon_1)\epsilon_0 + \frac{\sqrt{\epsilon_1}\sigma_{xx}(\omega)}{c} \right|^2} \quad \text{and} \quad R = \frac{\left| (\sqrt{\epsilon_1\epsilon_2} - \epsilon_1)\epsilon_0 + \sqrt{\epsilon_1}\frac{\sigma_{xx}(\omega)}{c} \right|^2}{\left| (\sqrt{\epsilon_1\epsilon_2} + \epsilon_1)\epsilon_0 + \sqrt{\epsilon_1}\frac{\sigma_{xx}(\omega)}{c} \right|^2}, \quad (50)$$

are plotted in figure 5. For zero chemical potential we see a constant behavior of the transmissivity and reflectivity in the visible region as expected due to the real part of the conductivity. For a finite chemical potential we see the influence of the threshold, visible due to $T = 1$ for $\hbar\omega < 2\mu$, which leads to a vanishing absorption. For $\hbar\omega > 2\mu$ we see also a constant behavior. This indicates, that for the given chemical potential the imaginary part, which is inversely proportional to the frequency, can be neglected for the visible region. With the graphene layer on a SiO_2 substrate we observe a reduction in the transmissivity and increase in the reflectivity. The behavior for $\omega = 0$ is not valid since we dropped the delta function in equation (32).

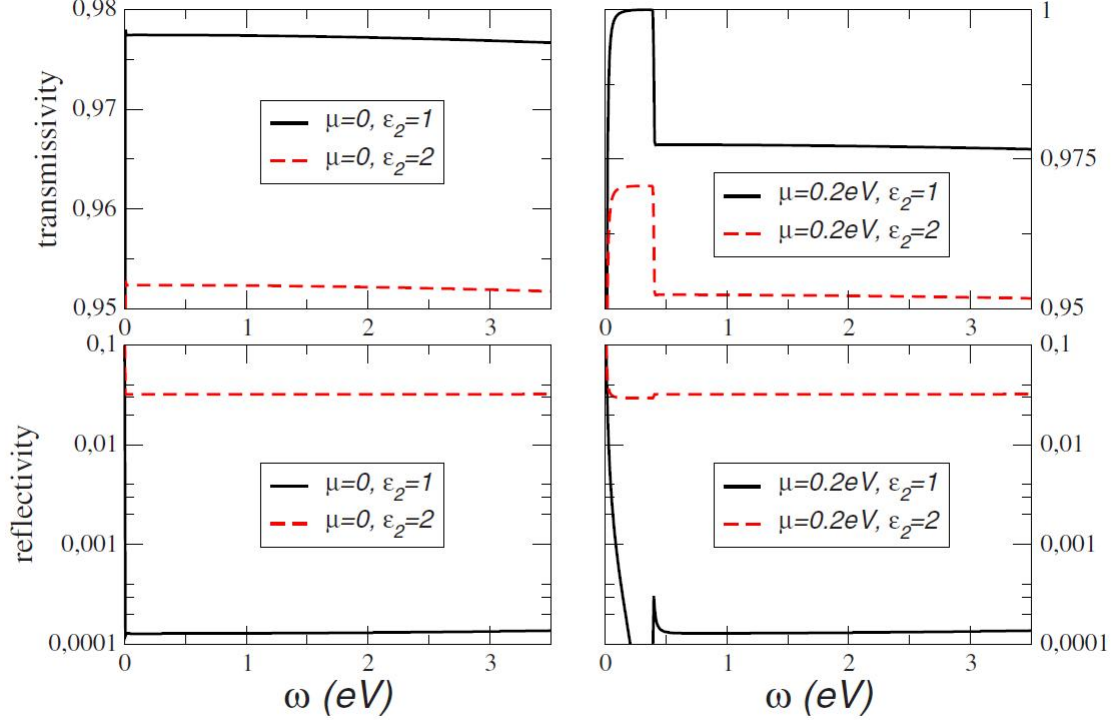


Figure 5: Transmissivity and reflectivity in the visible region for normal incidence and $T = 10\text{K}$, for the first medium vacuum ($\epsilon_1 = 1$) and the second medium is either vacuum ($\epsilon_2 = 1$) or a SiO_2 substrate ($\epsilon_2 = 2$) [1].

In the case given in equation (47), the transmissivity, reflectivity and absorption read

$$T = \frac{1}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \simeq 1 - \pi\alpha \approx 0.977, \quad (51)$$

$$R = \frac{\pi^2\alpha^2}{4}T \approx 0.00013, \quad (52)$$

and

$$A = 1 - T - R = \frac{\pi\alpha}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 0.022, \quad (53)$$

with the fine-structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0 c\hbar}. \quad (54)$$

The calculation is given the values $T \approx 0.977$, $R \approx 0.00013$ and $A \approx 0.022$. One recognizes, that all quantities are only dependent on universal constants and not on material properties, like e.g. the lattice constant. This is an exceptional behavior and can be used to calculate the fine structure by a simple transmission experiment. It is also interesting to see that a single layer of carbon atoms has for normal incidence an absorption of 2.2%. We can compare this results to the calculations made for bilayer graphene [2]. in the limit $\hbar\omega \gg t_\perp$, with t_\perp being the hopping parameter between the layers, the transmissivity of a bilayer graphene is given by

$$T \simeq 1 - 2\pi\alpha. \quad (55)$$

It is again only dependent on the fine-structure constant α and reduced by twice the value of a single layer. This theoretical prediction confirms the experimental results for the visual transparency of graphene [3] given in

figure 6, where a reduction in the transmittance of white light of 2.3% has been observed for a single graphene layer. Further, a stepwise reduction of $\pi\alpha$ for more layers was shown experimentally up to 4 layers. Additionally, the transmittance could be verified to be frequency independent in the visible region in good agreement with the theory.

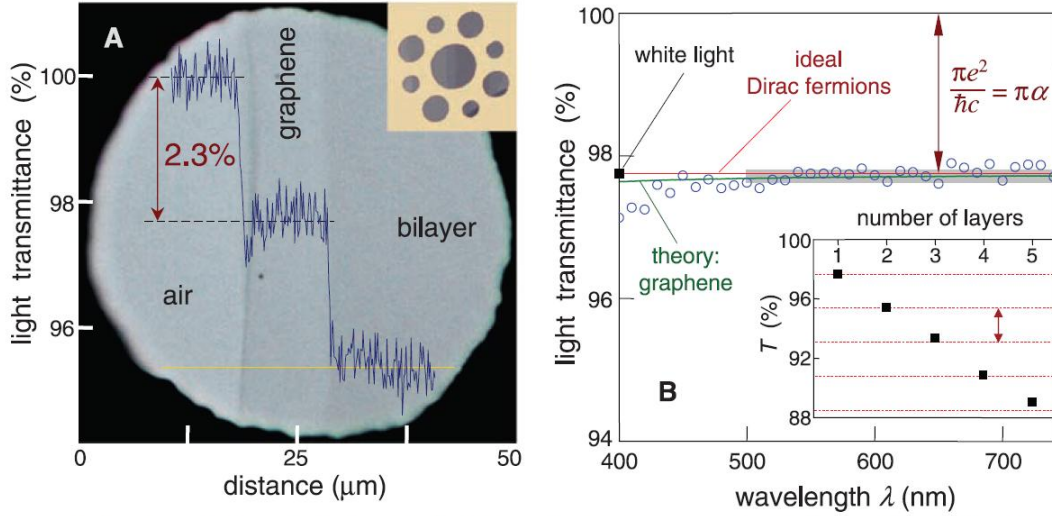


Fig. 1. Looking through one-atom-thick crystals. **(A)** Photograph of a 50- μm aperture partially covered by graphene and its bilayer. The line scan profile shows the intensity of transmitted white light along the yellow line. (Inset) Our sample design: A 20- μm -thick metal support structure has several apertures of 20, 30, and 50 μm in diameter with graphene crystallites placed over them. **(B)** Transmittance spectrum of single-layer graphene (open circles). Slightly lower transmittance for $\lambda < 500$ nm is probably due to hydrocarbon contamination (5). The red line is the transmittance $T = (1 + 0.5\pi\alpha)^{-2}$ expected for two-dimensional Dirac fermions, whereas the green curve takes into account a nonlinearity and triangular warping of graphene's electronic spectrum. The gray area indicates the standard error for our measurements (5). (Inset) Transmittance of white light as a function of the number of graphene layers (squares). The dashed lines correspond to an intensity reduction by $\pi\alpha$ with each added layer.

Figure 6: Experimental results for the visual transparency of graphene [3].

VI. CONCLUSIONS

We have found that the interaction of an electromagnetic field with a single graphene layer leads to an excitation of massless electrons at the K and K' point from valence to conduction band by absorbing the energy $\hbar\omega$ of a photon. In presence of a chemical potential, excitations with photon energies $\hbar\omega < 2\mu$ are suppressed due to the Pauli principle. The calculation of the optical conductivity showed that its real part vanishes for $\hbar\omega < 2\mu$ and remains nearly constant up to the end of the visible region of the spectrum. The imaginary part can be neglected for small chemical potentials and vanishes for $\mu = 0$. The transmissivity, reflectivity and absorption of a single layer or bilayer graphene are in the visible region of the spectrum only dependent on the fine structure constant α . The stepwise reduction in the transmittance of $\pi\alpha$ by increasing the number of layers is experimentally shown up to 4 layers and is in good agreement with the theoretical predictions for a single layer and bilayer graphene.

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